Calculus I

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Course Overview:
This course provides students with an opportunity to meet the following academic expectations:

- Speak clearly and communicate ideas accurately in a variety of settings
- Employ problem solving skills effectively
- Demonstrate critical thinking skills

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Course Units

Appendices

Key to Coding:
This curriculum document is aligned with the most recent Common Core State Standards for Mathematics.

The Common Core State Standards for Mathematics consist of six strands:

Number and Quantity, Algebra, Functions, Modeling*, Geometry, Statistics and Probability

*Making mathematical models is a Standard for Mathematical practice, and specific modeling standards appear throughout the high school standards. These standards are indicated by a star symbol (*) and apply to all standards in that group.

Number and Quantity

N-RN	The Real Number System
N-Q	Quantities*
N-CN	The Complex Number System
N-VM	Vector and Matrix Quantities

Algebra

A-SSE	Seeing Structure in Expressions
A-APR	Arithmetic with Polynomials and Rational Expressions
A-CED	Creating Equations*
A-REI	Reasoning with Equations and Inequalities

Functions

F-IF	Interpreting Functions
F-BF	Building Functions
F-LE	Linear, Quadratic, and Exponential Models*
F-TF	Trigonometric Functions

Geometry

G-CO	Congruence
G-SRT	Similarity, Right Triangles, and Trigonometry
G-C	Circles
G-GPE	Expressing Geometric Properties with Equations
G-GMD	Geometric Measurement and Dimension
G-MG	Modeling with Geometry

Statistics and Probability

S-ID	Interpreting Categorical and Quantitative Data
S-IC	Making Inferences and Justifying Conclusions
S-CP	Conditional Probability and the Rules of Probability
S-MD	Using Probability to Make Decisions

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Unit 1: Functions and Models

Abstract

Functions are fundamental to our study of calculus. In this unit we explore the basic concepts about functions: how to combine and transform functions, and how to interpret characteristics of their graphs. Functions are tools used for modeling mathematics and will be examined numerically, graphically and algebraically, throughout this course. We examine real-world applications for the main types of functions. Some exploration will also be done with the graphing calculator.

Essential Questions: How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?

Focus Questions:

1. What is a function and describe its characteristics?

2. How and why are function groups referred to as "families"?

3. How can real-world situations be represented using functions?

Benchmarks:

The student will be able to

1. determine whether a relation represents a function by using the formal definition of function and by viewing the associated graph.

2. evaluate functions for given values and express domain and range using interval notation.

3. graph functions and identify characteristics such as end-behavior and asymptotes, intervals of increase/decrease, maximums/minimums, concavity and inflection points

4. identify and sketch families of graphs.

5. identify trigonometric functions and solve application problems.

6. evaluate logarithmic and exponential functions.

Common Core State Standards for Mathematics:
Functions
F-IF 1, 2, 4*, 5*, 6*, 7a*, 7c*, 7d*, 7e*
F-LE* 1a, 1b,1c, 3
F-TF 5*, 7*

Technology Education Framework Connection:
Content Standards
Calculators: graphing utility

Suggested Activities: *Instructor's Resource Center (IRC)*

Identical Unit Assessment:

Teacher generated summative assessment that is specifically aligned to unit benchmarks.

Instructional Resources and Materials:

Graphing Calculator (TI-83 or TI-84)
Single Variable Calculus: Concepts and Contexts, James Stewart, 2010; **Chapter 1**

Pacing: This unit is expected to take approximately ten, 80-minute class periods.

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Unit 2: Limits and Derivatives

Abstract

Limits occur naturally in everyday events and are fundamental to the study of calculus. In this unit, we investigate limits and their properties. This gives rise to a major focus in calculus, the derivative. Students will use the limit definition of the derivative to find rates of change when analyzing functional behavior. Specifically, students will learn about slopes of tangents to a function at a specific point and velocities. Knowing the derivative of a function can provide valuable information about the original form of the function.

<u>Essential Question:</u> How do patterns and functions help us describe data and physical phenomena and solve a variety of problems?

<u>Focus Questions:</u>

1. How can limits be used to investigate the global behavior of functions?

2. How can limits be applied to real world applications?

3. How can limits be used to define the derivative?

<u>Benchmarks:</u>

The student will be able to

1. understand the concept of limits and be able to explore limits by table and graphically.

2. compute limits using algebra.

3. apply the concept of continuity and its implications, including the Intermediate Value Theorem.

4. calculate limits involving infinity.

5. calculate the average rate of change using tables and graphically.

6. define the derivative using the limit definition and use the definition to find rates of change involving tangents and velocities.*

7. use the limit definition of derivative to show the relationship between differentiability and continuity.

8. given the graph of a function, sketch the graph of its derivative, and vice versa.

9. sketch an original function given knowledge about the function's first and second derivatives.

Common Core State Standards for Mathematics:
Functions
F-IF 4*, 6*, 7*

Technology Education Framework Connection:
Content Standards
Calculators: graphing utility

Suggested Activities: *Instructor's Resource Center (IRC)*

CBL Activity Velocity Test: Verifying Velocity from Real World Math with the CBL System, 1999
Use the Connecticut State Income Tax Booklet to determine if the tax formulas represent a continuous (ie. Fairly taxed) function.
Match graphs of functions and their derivatives.

Identical Unit Assessment:

Teacher generated summative assessment that is specifically aligned to unit benchmarks.

Instructional Resources and Materials:
Graphing Calculator (TI-83 or TI-84)
Single Variable Calculus: Concepts and Contexts, James Stewart, 2010; **Chapter 2**

Pacing: This unit is expected to take approximately twenty, 80-minute classes.

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Unit 3: Differentiation Rules

Abstract

We have learned that a derivative can be interpreted as the slope of a tangent line to a curve - a rate of change. We have estimated the derivative from tables of values, from graphs, and by formula using a special definition. But the definition can be long and tedious. In this unit, we develop rules for finding derivatives indirectly. This enables us to calculate derivatives of polynomials, rational functions, algebraic functions, exponential and logarithmic functions, trigonometric and inverse functions with ease. We continue to solve problems involving rates of change and tangents.

Essential Question: How do numbers represent quantitative relationships?

Focus Questions:

1. What are the derivative rules and how are they applied?

2. How are real-world applications solved with the use of derivatives?

Benchmarks:

The student will be able to:

1. compute derivatives using the power, sum/difference, product and quotient rules.
2. compute derivatives of trigonometric functions.
3. compute derivatives using the chain rule.
4. use implicit differentiation to calculate derivatives.
5. calculate derivatives of logarithmic and exponential functions.
6. use derivatives to solve real-world problems, such as particle and projectile motion problems.*

Common Core State Standards for Mathematics:
Functions
F-IF 6*, 7*

Technology Education Framework Connection:
Content Standards
Calculators: graphing utility

Suggested Activities:

Identical Unit Assessment:

Teacher generated summative assessment that is specifically aligned to unit benchmarks.

Instructional Resources and Materials:

Graphing Calculator
Single Variable Calculus: Concepts and Contexts, James Stewart, 2010; **Chapter 3**

Pacing: This unit is expected to take approximately fifteen, 80-minute classes.

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Unit 4: Applications of Differentiation

Abstract

With the arsenal of differentiation rules at our disposal, we can pursue applications of differentiation in greater depth. We learn how to analyze behavior of families of functions, how to solve related rates problems, how to easily find the maximum or minimum values of a function, and how to solve optimization problems. At the end of the unit we investigate working backwards from the derivative by exploring the concept of anti-derivatives.

Essential Question: How do numbers represent quantitative relationships?

Focus Questions:

1. How are real-world related rate application problems solved using derivatives?

2. How are functions graphed using first and second derivative tests?

3. How are real world optimization application problems solved using derivatives?

Benchmarks:

The student will be able to

1. use derivatives to solve related rate applications.*

2. calculate maximum/minimum values and determine intervals of increase/decrease using the first derivative test.*

3. using the second derivative test calculate concavity and points of inflection.

4. graph a function using the results of the first and second derivative tests.

5. investigate indeterminate forms and l'Hospital's Rule.

6. use derivatives to solve optimization applications.*

Common Core State Standards for Mathematics:
Functions
F-IF 4*, 6*, 7*

Technology Education Framework Connection:
Content Standards
Calculators: graphing utility

Suggested Activities: *Instructor's Resource Center (IRC)*

Identical Unit Assessment:

Teacher generated summative assessment that is specifically aligned to unit benchmarks.

Instructional Resources and Materials:

Graphing Calculator (TI-83 or TI-84)

Single Variable Calculus: Concepts and Contexts, James Stewart, 2010; **Chapter 4**

Pacing: This unit is expected to take approximately fifteen, 80-minute classes.

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Unit 5: Integrals

Abstract

In differentiable calculus, when given a function we can find its derivative. In integral calculus, given the derivative we can find the function from which it was derived. We begin with the student's basic understanding about distance and rectangular area, then apply knowledge about limits to formulate the concept of the definite integral, i.e. the area under a curve from *a* to *b*. The Fundamental Theorem of Calculus relates the integral to the derivative and vice versa, therefore tying together two branches of calculus. This simplifies the solution to many problems and real-world applications. In this unit, we also learn other specialized techniques for integration, such as substitution and integration by parts.

Essential Question: How do numbers represent quantitative relationships?

Focus Questions:

1. How can areas be calculated using approximation methods?
2. What is the relationship between derivatives and integrals?
3. How are the rules for integration used to solve problems?

Benchmarks:

The student will be able to (in both radians and degrees)

1. determine antiderivatives of functions.
2. calculate areas and distance graphically and using Riemann summation method.
3. evaluate the definite integral using the Fundamental Theorem of Calculus Parts I and II.
4. calculate integrals using the substitution method.

Common Core State Standards for Mathematics:
Functions
F-IF 4, 6, 7

Technology Education Framework Connection:
Content Standards
Calculators: graphing utility

Suggested Activities

Identical Unit Assessment:

Teacher generated summative assessment that is specifically aligned to unit benchmarks.

Instructional Resources and Materials:

Graphing Calculator (TI-83 or TI-84)
Single Variable Calculus: Concepts and Contexts, James Stewart, 2010; **Chapter 5**

Pacing: This unit is expected to take approximately fifteen, 80-minute classes.

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Unit 6: Applications of Integration

Abstract

In this unit students learn how to apply their knowledge and skill of calculating integrals to problem solving. Students learn how to use integration calculations to determine such things as areas bound between curves and volumes of solids. Volume problems involve the rotation of a defined area about an axis, creating solids such as rings, discs and washers.

<u>Essential Question:</u> How do numbers represent quantitative relationships?

<u>Focus Questions:</u>

1. How can areas between curves be calculated using definite integrals?

2. How can volumes of solids be calculated using definite integrals?

<u>Benchmarks:</u>

The student will be able to

1. calculate the area under the curve of a function using integration.

2. calculate the area between two curves using top/bottom and right/left methods of integration.

3. calculate volume of solid shapes rotated about the horizontal and vertical axes using the circular disc method of integration.*

4. calculate the volume of nonsolid shapes using the washer method of integration.*

Common Core State Standards for Mathematics:
Functions
F-IF 4*, 7*

Technology Education Framework Connection:
Content Standards
Calculators: graphing utility

Suggested Activities:

Identical Unit Assessment:

Teacher generated summative assessment that is specifically aligned to unit benchmarks.

Instructional Resources and Materials:

Graphing Calculator (TI-83 or TI-84)
Single Variable Calculus: Concepts and Contexts, James Stewart, 2010; **Chapter 6**

Pacing: This unit is expected to take approximately ten, 80-minute classes.