

# Calculus BC

2007 - 2008

---

## Brief Description of Course

This course covers the topics limits, continuity, differentiation, antidifferentiation, definite integrals, techniques of integration, series and sequences, parametric and polar functions, and vectors, with applications to the physical and engineering sciences.

## Unit Information

### **Unit Name or Timeframe:**

Limits and Continuity: Chapter 1

2 weeks

### **Content and/or Skills Taught:**

- Concept of limit, exploring limits graphically using the graphing calculator, which will include conjecturing the limit from the graph and estimating limits numerically using the table feature (includes one-sided limits)
- Computation of limits using algebra
- Concept of continuity and its implications (Intermediate Value Theorem, explain in writing and sketch examples of types of functions that have essential vs. removable discontinuity (holes, jumps, asymptotes)
- Limits Involving Infinity (asymptotic and unbounded behavior)
- Formal Definition of Limit (delta-epsilon)

### **Major Assignments and/or Assessments:**

### **Unit Name or Timeframe:**

Differentiation: Chapter 2

3 weeks

### **Content and/or Skills Taught:**

- Concept of Derivative: Tangent Lines, Instantaneous Rate of Change, Velocity (investigate the concept of the derivative with the graphing calculator by estimating the value of the derivative using the graph and table feature to calculate the average rate of change over increasingly smaller intervals)
- Limit Definition of the Derivative (limit of the difference quotient, limit of the average rate of change), relationship between differentiability and continuity
- Computation of Derivatives: Power, Sum/Difference, Product and Quotient Rules
- Derivatives of Trigonometric Functions
- Derivatives of Exponential and Logarithmic Functions
- Chain Rule
- Implicit Differentiation and Related Rates
- Mean Value Theorem (including the geometric meaning, which can be shown using the graphing calculator)

**Major Assignments and/or Assessments:**

CBL Velocity Test: Interpreting Velocity Graphs

How Many Licks? Related Rate Activity using Tootsie Pops

Both of these activities require the use of the graphing calculator, and students are required to submit a write-up of their findings.

**Unit Name or Timeframe:**

Applications of Differentiation: Chapter 3

2 weeks

**Content and/or Skills Taught:**

-Linear Approximation and L'Hopital's Rule

-Newton's Method

-Maximum and Minimum Values (First, use the graphing calculator to identify the location of the extrema as the "peaks" and "troughs" of the graph to lead students to a conjecture that such values can be found independent of the graph by having the slope, ie, first derivative, equal to zero and changing sign. Then, formally state the Derivative Test. Also, use the graph to compare the meanings of relative vs. absolute extrema for a single function, then formally state the Extreme Value Theorem)

-Increasing and Decreasing Functions (First, use the graphing calculator to investigate the relationship between the increasing and decreasing behavior of  $f$  and the sign of  $f'$ , express verbally and in writing corresponding characteristics of graphs of  $f$  and  $f'$  as conclusion of such investigation)

-Concavity (points of inflection as places where concavity changes; investigate using the graphing calculator the relationship between the concavity of  $f$  and the sign of  $f''$  and express verbally and in writing the corresponding characteristics of graphs of  $f$  and  $f''$  as conclusion of such investigation; formally state the Second Derivative Test)

-Overview of Curve Sketching (students will synthesize the corresponding characteristics of the graphs of  $f$ ,  $f'$ , and  $f''$  by graphing all three functions on a single plane and making conclusions in applied contexts, including velocity, speed and acceleration)-Optimization as a real-world application of the absolute extrema, where students must show the calculations leading to their conclusion, which must be communicated verbally.

**Major Assignments and/or Assessments:**

Sample AP Free-Response Exam Questions, as applicable

For free-response questions that require the use of the graphing calculator, students must give a written justification of their answer as prompted.

**Unit Name or Timeframe:**

Integration: Chapter 4

3 weeks

**Content and/or Skills Taught:**

-Antiderivatives (antiderivatives following directly from derivatives of basic functions, determine a position function given the velocity or acceleration function and an initial condition)

-Sums and Sigma Notation

-Area (calculating area using left, right, and midpoint evaluation of Riemann sums, using limit of a sum to calculate the exact area)

-The Definite Integral (interpretation of the definite integral as a limit of Riemann sums over subdivisions, basic properties of definite integrals, interpretation of the definite integral as the net change of a quantity over an interval, net change of distance/signed area versus total distance/total area, average value of a function, Integral Mean Value Theorem)

-The Fundamental Theorem of Calculus Parts 1 and 2 (use of part 1 of the FTC to evaluate definite

integrals, use of part 2 of the FTC to find the derivative of a function defined by an integral and the analytical and graphical analysis of such a function)

-Integration by Substitution ( $u$ ,  $du$  including change of limits for definite integrals)

-Numerical Integration (use of Riemann and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by a table of values; using the graphing calculator and the table feature to generate values of the function)

**Major Assignments and/or Assessments:**

CBL Walk This Way Activity: Introduction to Integration

This activity requires the use of the graphing calculator, and students are required to submit a write-up of their findings.

Sample AP Free-Response Exam Questions, as applicable

For free-response questions that require the use of the graphing calculator, students must give a written justification of their answer as prompted.

**Unit Name or Timeframe:**

Application of the Definite Integral: Chapter 5

2 weeks

**Content and/or Skills Taught:**

-Area Between Two Curves (top/bottom, right/left,  $dx$  and  $dy$ , both by hand and by the graphing calculator)

-Volume by Circular Disk Method (horizontal and vertical axes of revolution, both by hand and by the graphing calculator)

-Volume by Circular Washer Method (horizontal and vertical axes of revolution, both by hand and by the graphing calculator)

-Volume by Slicing (cross-sectional areas perpendicular to the  $x$ -axis)

-Arc Length and Surface Area

**Major Assignments and/or Assessments:**

Play-Doh Activity (acquired from an AP workshop) to assist in visually demonstrating volumes by slicing.

Sample AP Exam Free-Response Questions, as applicable

For free-response questions that require the use of the graphing calculator, students must give a written justification of their answer as prompted.

**Unit Name or Timeframe:**

Exponentials, Logarithms, and Other Transcendental Functions: Chapter 6

3 weeks

**Content and/or Skills Taught:**

-The Natural Logarithm Revisited (redefine  $\ln x$  as the definite integral of  $1/t$  from 1 to  $x$ ; review of logarithm properties; derivatives of logs and integrals of functions in the form of  $1/x$ )

-Inverse Functions (use the graphing calculator to conjecture the geometric relationship between a function and its inverse; how to find an inverse algebraically, if possible; the connection between one-to-one and continuity, using the first derivative to determine if the function is monotonic, thus one-to-one; calculating the derivative of an inverse of a function indirectly)

-The Exponential Function Revisited (redefine  $e^x$  and review of derivatives and integrals involving exponential functions with any base  $b$ )

-Growth and Decay Problems (deriving exponential models, including Newton's Law of Cooling, to solve real-world problems involving growth and decay with written or verbal summaries where

applicable)

- Separable Differential Equations (solving separable differential equations using an initial condition, solving logistic differential equations and using them in modeling)
- Slope Fields (constructing slope fields given a differential equation, draw a specific solution given an initial condition both with and without a graphing calculator)
- Euler's Method (using the table feature of the graphing calculator find the numerical solutions of differential equations compare this answer with the solution acquired using Euler's Method)
- Inverse Trigonometric Functions (review of definitions from pre-calculus)
- The Calculus of Inverse Trigonometric Functions (derivatives and integrals of the inverse trigonometric functions)
- Hyperbolic Functions (definitions, identities, derivatives, and integrals involving the hyperbolic trigonometric functions and their inverses)

**Major Assignments and/or Assessments:**

A Watched Cup Never Cools: A Newton's Law Activity or CBL Chill Out: How Hot Objects Cool Extension Activity. A 2-3 page written response with attached calculations is required.

AP 2004-2005 Workshop Material Activities with Differential Equations and Slope Fields

Sample AP Free-Response Exam Questions, as applicable

For free-response questions that require the use of the graphing calculator, students must give a written justification of their answer as prompted.

**Unit Name or Timeframe:**

Integration Techniques: Chapter 7

3 weeks

**Content and/or Skills Taught:**

- Integration by Parts (have the students verbally discuss their best strategy for determining which part of the integrand should be  $u$  and which part should be  $dv$ )
- Trigonometric Techniques of Integration
- Integration of Rational Functions Using Partial Fraction Decomposition
- Integration Tables and Computer-Algebra Systems
- Indeterminate Forms (as limits of definite integrals) and L'Hopital's Rule
- Improper Integrals (including using L'Hopital's rule to determine limits and convergence) Students will use the comparison test and tables generated by using a graphing calculator to determine the convergence or divergence of an improper integral.

**Major Assignments and/or Assessments:**

Sample AP Free-Response Exam Questions, as applicable

For free-response questions that require the use of the graphing calculator, students must give a written justification of their answer as prompted.

**Unit Name or Timeframe:**

Infinite Series: Chapter 8

3 weeks

**Content and/or Skills Taught:**

- Sequences of Real Numbers (definitions, limits, convergence and divergence, properties, boundedness, monotonicity; also investigate through the use of the table feature of the graphing calculator the convergence of sequences that cannot be directly computed)
- Infinite Series (geometric series, harmonic series; also investigate through the use of the table feature of the graphing calculator the convergence of series that cannot be directly computed)

- Integral Test (using terms of series as areas of rectangles as related to an improper integral to determine convergence, convergence of p-series)
- Comparison Tests (comparing a given series with a known series to determine convergence)
- Alternating Series (with error bound)
- Ratio Test (for absolute convergence and divergence, Root Test)
- Power Series (including functions defined by power series, finding radius of convergence)
- Taylor Series (including using Taylor polynomials to approximate curves, Maclaurin series as a special case, Lagrange error bound)
- Applications of Taylor Series

**Major Assignments and/or Assessments:**

Sample AP Free-Response Exam Questions, as applicable

For free-response questions that require the use of the graphing calculator, students must give a written justification of their answer as prompted.

**Unit Name or Timeframe:**

Parametric, Vector, and Polar Functions: Chapter 9 plus  
2 weeks

**Content and/or Skills Taught:**

- Plane Curves and Parametric Equations (sketch graphs of plane curves defined by parametric equations both by hand and using the graphing calculator, have students verbalize the connection between the parametric equations and their corresponding rectangular equation)
- Calculus and Parametric Equations (derivatives/velocity, integrals/area under curve in parametric form, analysis)
- Arc Length and Surface Area in Parametric Equations
- Polar Coordinates (have students give examples of graphs which are easier in polar rather than rectangular form)
- Calculus and Polar Coordinates (slope, horizontal and vertical tangent lines; derivatives and integrals/area of region bound by polar curves, analysis)
- Vectors (angle between vectors, scalar product, and using vectors to describe motion in the plane with derivatives and integrals of vector functions)

**Major Assignments and/or Assessments:**

Sample AP Free-Response Exam Questions, as applicable

For free-response questions that require the use of the graphing calculator, students must give a written justification of their answer as prompted.

Graphing Calculator Lab in which students investigate the effect of adding or multiplying by a constant to polar equations, and communicate both verbally and in writing their observations.

Design a Scrambler Project

Use of a graphing calculator and a written report are part of this lab's requirements.

**Textbooks**

**Title:**Calculus

**Publisher:** McGraw Hill

**Published Date:** 2002

**Author:** Robert T. Smith

**Second Author:** Roland B. Minton

**Description:**

Second Edition, Single Variable

**Other Course Materials**

**Material Type:**Graphing Calculator

**Description:**

TI-83 Plus and TI-89

**Material Type:**Other

**Description:**

- A Watched Cup Never Cools by Ellen Kamischke
- Real World Math with the CBL 2 and LabPro: Activities for TI-83 Plus by Chris Brueningsen, et. al.
- Advanced Placement Program Professional Development for Calculus: Workshop Materials
- Calculus, Second Edition, Multi-Variable by Smith and Minton, 2002
- Preparing for the (AB) AP Calculus Examination by George W. Best and Richard J. Lux
- Preparing for the (BC) AP Calculus Examination by George W. Best and Richard J. Lux

**Websites**

**URL:**[www.apcentral.collegeboard.com](http://www.apcentral.collegeboard.com)

**Description:**

This website is a source for AP exam questions from previous years.